

**NASA  
Technical  
Paper  
2310**

**July 1984**

NASA-TP-2310 19840019475

**A New Method for  
Determining Acoustic-Liner  
Admittance in a Rectangular  
Duct With Grazing Flow  
From Experimental Data**

**Willie R. Watson**

**NASA**



**NASA  
Technical  
Paper  
2310**

1984

A New Method for  
Determining Acoustic-Liner  
Admittance in a Rectangular  
Duct With Grazing Flow  
From Experimental Data

Willie R. Watson

*Langley Research Center  
Hampton, Virginia*



National Aeronautics  
and Space Administration

Scientific and Technical  
Information Branch



## INTRODUCTION

The acoustic admittance of practical liners may be a function of the aeroacoustic environment in which they are located. This fact necessitates that their properties be evaluated in such an environment. This is accomplished in the laboratory with a grazing-flow impedance tube (or rectangular duct). In such a tube, the acoustic material is aligned so that the sound and mean flow graze over the surface of the material in a controlled manner to simulate the environment typical of that found in a real jet engine. Since admittance is defined as the ratio of normal particle velocity to pressure at the wall surface, the admittance could be determined experimentally by direct measurements of normal particle velocity and pressure. In practice, measurements of normal velocity in the presence of grazing flow are beyond present measurement techniques. Instead, an indirect approach must be used which depends only on the measurement of acoustic pressure at selected locations in the test configuration. These measurements are then used as input to an analytical program which determines the admittance based upon these measured data.

To date, two methods have been presented for determining the acoustic admittance in grazing flow from measured values of the axial propagation constant and cross modes. The first method is that described by Mungur and Gladwell (ref. 1). This method is applicable to general flows and is based upon a Runge-Kutta integration technique. The second method is described by Armstrong et al. (ref. 2). Armstrong's method is restricted to thin boundary-layer flows. This method uses an asymptotic expansion to derive a boundary condition that is applicable at the boundaries of the uniform region, in which an exact solution is possible.

Both methods presented in references 1 and 2 are based upon the solution to an ordinary differential equation and are restricted to mean flows with transverse gradients in one direction only. However, grazing-flow duct facilities commonly employ ducts with a relatively small rectangular cross section in which the grazing flow possesses gradients in both transverse directions of the impedance tube. Such is the case here in the flow impedance test laboratory at the Langley Research Center. Thus, the present effort was motivated by the need to account for the more realistic flow environment in laboratory flow-impedance tubes such as those in the Langley facility.

This paper describes a new method for determining the admittance of an acoustic material in a grazing-flow impedance tube (or duct). The method is developed explicitly for rectangular ducts, although it is applicable to other duct geometries as well. The approach is to obtain an estimate of the axial propagation constant, cross-mode order, and mean-flow profile by means of acoustic and aerodynamic measurements. These experimentally determined characteristics of the aeroacoustic field are then input into an analytical program from which the admittance is calculated. Unlike the methods employed in references 1 and 2, the approach presented in this research is extendable to include mean flows with gradients in both transverse directions of an impedance tube. This paper gives a detailed description of the method employed when the mean flow is restricted to gradients in only one transverse direction. A formulation of the method to include mean-flow gradients in both transverse directions will be reserved for a future paper.

The present paper is divided into three sections. The first section presents the governing differential equation, boundary conditions, and a description of the measured data necessary for a determination of the admittance. The second section describes the numerical method employed to determine the admittance. The paper closes with a results section (third section) in which admittance values determined by this method are compared both with exact values that can be obtained for a constant mean-flow profile and with results from a Runge-Kutta integration technique for cases involving a one-dimensional boundary layer. Such comparisons serve as confidence builders giving credibility to the method.

## SYMBOLS

$\left. \begin{matrix} [A(\beta_{w,u})], [B], [D] \\ [\bar{B}], [\bar{D}] \end{matrix} \right\}$  global matrices

$[A^q]$  stiffness matrix for element  $q$

$\left. \begin{matrix} a_{11}^q, a_{12}^q, a_{21}^q \\ a_{22}^q, b_{I,J} \\ b_{N,N}, d_{N,N}, \bar{b}_{N,N}, \bar{d}_{N,N} \end{matrix} \right\}$  matrix elements

$b$  dimension of line element

$c$  ambient speed of sound

$E(y)$  residual error

$F(y)$  one-dimensional acoustic pressure eigenfunction

$F_I, M_I$  respective values of functions  $F(y)$  and  $M(y)$  at node  $I$

$f_I(y, b)$  element shape function

$h$  duct height

$i = \sqrt{-1}$

$K$  free-space wave number

$K_x$  dimensional axial propagation constant

$\tilde{K}_x$  dimensionless axial propagation constant

$L$  duct width

$M(y)$  Mach number of mean-flow profile

$M_0$  centerline Mach number

$m$	cross-mode order
$N$	total number of locations in transverse direction of impedance tube
$n$	boundary-condition parameter
$P(y,z)$	acoustic pressure eigenfunction in impedance-tube cross section
$t$	time
$x,y,z$	rectangular coordinates
$\bar{y}$	local coordinate for element
$y_I$	value of global coordinate $y$ at node $I$
$\beta_{w,u}, \beta_{w,l}$	specific acoustic admittance of upper and lower walls, respectively, in impedance tube
$\lambda$	eigenvalue
$\{\Phi\}$	global vector of unknowns
$\{\Phi^q\}$	vector of unknown nodal values for element $q$
$\omega$	angular frequency
Subscripts:	
$I, J$	integers
$N$	number
Superscripts:	
$n$	integer (1 or 2)
$q$	element number
$Tr$	transpose
Mathematical notation:	
$[ \ ]$	denotes matrix
$\{ \}$	denotes vector
$'$	first derivative
$''$	second derivative

## PROBLEM FORMULATION

Figure 1 depicts the cross-section geometry of the grazing-flow impedance-tube (rectangular-duct) test section and its coordinate system. The mathematical

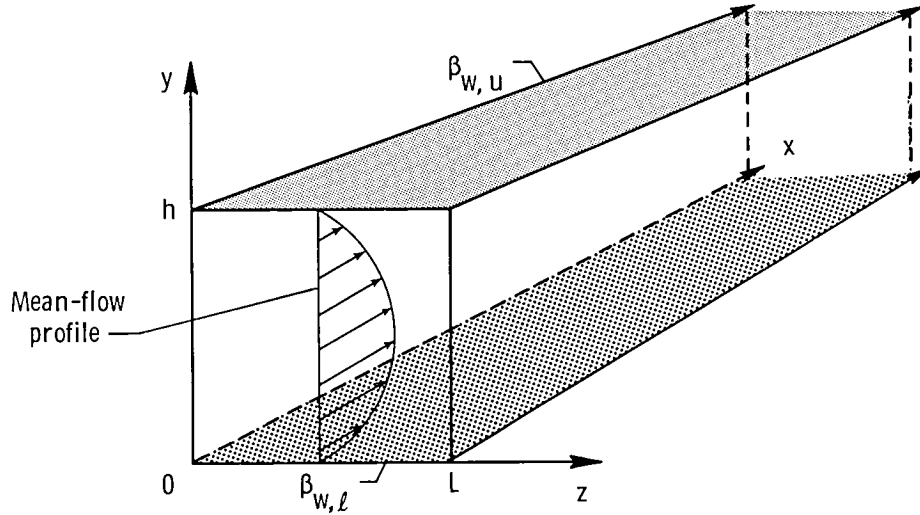


Figure 1.- Grazing-flow impedance-tube test section and coordinate system.

development assumes that the mean flow in the test section is axial and fully developed so that it is independent of the axial coordinate  $x$ . The two sidewalls of the tube are assumed rigid, whereas the upper and lower walls are lined with acoustic material. The specific acoustic admittances of the upper and lower walls are  $\beta_{w,u}$  and  $\beta_{w,l}$ , respectively. All four walls in the figure are assumed to be of infinite extent in the axial direction (infinite-length assumption), and sound waves are assumed to travel in the axial direction. Further, the mean flow is assumed to have an axial component only and flow gradients are assumed to exist only in the  $y$ -direction of the tube.

Upon taking into account the fundamental equations of fluid dynamics (i.e., the momentum equations and the continuity equation) and assuming that the acoustic pressure field is of the form  $P(y,z) e^{-i(K_x x - \omega t)}$ , the following elliptic partial differential equation for the pressure eigenfunction  $P(y,z)$  is obtained (ref. 3):

$$P_{yy} + P_{zz} + \frac{2\tilde{K}_x M_x P_y}{1 - M_x^2} + K^2 \left[ \left( 1 - M_x^2 \right)^2 - \tilde{K}_x^2 \right] P = 0 \quad (1)$$

where

$$\tilde{K}_x = K_x / K$$



In equation (1) the function  $M(y)$  is the Mach number of the mean-flow profile,  $\tilde{K}_x$  is the dimensionless axial propagation constant in the tube, and  $K$  is the free-space wave number  $\omega/c$ .

Boundary conditions associated with equation (1) are expressed in the form

$$P_z(y, 0) = 0 \quad (2)$$

$$P_z(y, L) = 0 \quad (3)$$

$$P_y(0, z) = -iK\beta_{w,\lambda} \left[ 1 - M(0) \tilde{K}_x \right]^n P(0, z) \quad (4)$$

$$P_y(h, z) = iK\beta_{w,u} \left[ 1 - M(h) \tilde{K}_x \right]^n P(h, z) \quad (5)$$

Continuity of acoustic-particle displacement at the lined walls is obtained by setting the integer  $n$  to 2, whereas continuity of acoustic-particle velocity is obtained by setting  $n$  to 1. In recent years, continuity of acoustic-particle displacement has been more universally accepted as the correct boundary condition; however, realistic mean-flow profiles satisfy the condition of no slip at the boundaries and render a zero mean flow there. Thus, for realistic mean flows, both forms of the boundary condition coalesce to the same expression at each wall. However, to preserve the generality, both forms of the boundary condition will be incorporated in this paper by carrying  $n$  throughout the analysis as a parameter.

The solution to equations (1) through (5) can be expressed in the form

$$P(y, z) = F(y) \cos(m\pi z/L) \quad (m = 0, 1, 2, 3, \dots) \quad (6)$$

where the eigenfunction  $F(y)$  satisfies the ordinary differential equation

$$F'' + \left( 2\tilde{K}_x M' F' \right) / \left( 1 - M\tilde{K}_x \right) + K^2 \left\{ \left( 1 - M\tilde{K}_x \right)^2 - \left[ \tilde{K}_x^2 + (m\pi/KL)^2 \right] \right\} F = 0 \quad (7)$$

and the boundary conditions

$$F'(0) = -iK\beta_{w,\lambda} \left[ 1 - M(0) \tilde{K}_x \right]^n F(0) \quad (8)$$

$$F'(h) = iK\beta_{w,u} \left[ 1 - M(h) \tilde{K}_x \right]^n F(h) \quad (9)$$

Equations (6) through (9) are easily derived by finding the general solution to equation (1) by separation of variables and then applying the boundary conditions (equations (2) through (5)).

Equations (7) through (9) constitute a boundary-value problem for the acoustic pressure eigenfunction  $F(y)$ . The wave number  $K$ , mean-flow profile  $M(y)$ , and boundary-condition parameter  $n$  in these equations are specified and are treated as known. The four remaining parameters of this problem are the axial propagation constant  $\tilde{K}_x$ , the cross-mode order  $m$ , and the admittances of the wall lining  $\beta_{w,u}$  and  $\beta_{w,l}$ . Only three of these parameters are independent and can be specified. The fourth parameter is then determined so that the governing differential equation and boundary conditions are satisfied. Generally, when given the admittances  $\beta_{w,u}$  and  $\beta_{w,l}$  and cross-mode order  $m$ , a corresponding set of axial propagation constants  $\tilde{K}_x$  are determined. This paper focuses on the inverse problem. Thus, the admittance  $\beta_{w,l}$  will be specified,  $\tilde{K}_x$  and  $m$  are assumed to be obtained from acoustic measurements, and the admittance  $\beta_{w,u}$  will be determined. Although an infinite set of axial propagation constants  $\tilde{K}_x$  exist for each cross-mode order  $m$ , it is assumed that one member of this set can be measured.

Upon obtaining the value of the necessary parameters, the boundary-value problem posed by equations (7) through (9) can be solved. However, equation (7) is an ordinary differential equation with variable coefficients, and its solution can be put in terms of known functions only for some special cases of  $M$ . Therefore, these cases are not useful for general application. Thus, the admittance must be determined numerically for mean flows of practical interest, and this paper develops a numerical method for calculating this admittance. The numerical method is based upon a Galerkin finite-element method and is described in the following section.

#### NUMERICAL METHOD

In the Galerkin finite-element method employed here, the transverse direction of the impedance tube (rectangular duct) is divided into a total of  $N-1$  line elements, each with dimension  $b$  as shown in figure 2. These elements are considered

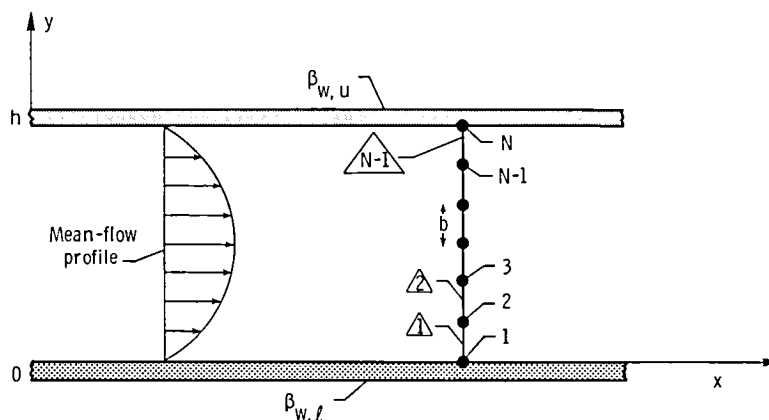


Figure 2.- Finite-element discretization and numbering system for grazing-flow impedance-tube test section.

interconnected at joints which are called the nodes of the system. The nodes of the system are designated by the dark circles in the figure and are numbered appropriately. The number for each element is enclosed in a triangle and is identical with the lower node number of the element. In order to generalize and also simplify the formulation, it is convenient to define a local coordinate system for each element as shown in figure 3. Thus, global coordinates  $y_I$  and  $y_{I+1}$  for a typical element  $I$  of the global system are referred to as nodes 1 and 2, respectively, in this local coordinate system. Not only does this local coordinate system generalize and simplify the formulation but it will also facilitate the integration which is required to obtain the element equations.

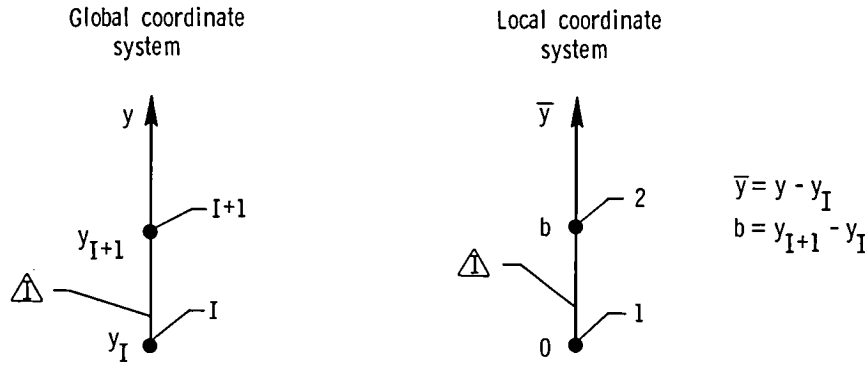


Figure 3.- Global and local coordinate systems for typical finite element.

In this paper, a linear finite-element method is used. Thus, in each element the eigenfunction  $F(y)$  and Mach number  $M(y)$  are expressed in a form which ensures continuity of the functions  $F(y)$  and  $M(y)$  at the nodes joining two elements. In a typical element, these functions are expressed in the form

$$F(\bar{y}) = f_I(\bar{y}, b) F_I \quad (I = 1, 2) \quad (10)$$

$$M(\bar{y}) = f_I(\bar{y}, b) M_I \quad (I = 1, 2) \quad (11)$$

in which repeated subscripts are to be summed over,  $F_I$  and  $M_I$  are the values of the functions  $F(y)$  and  $M(y)$ , respectively, at node  $I$  of the element, and

$$f_1(\bar{y}, b) = 1 - (\bar{y}/b) \quad (12)$$

$$f_2(\bar{y}, b) = \bar{y}/b \quad (13)$$

In following the method of Galerkin, the residual error  $E(y)$  in each element is multiplied by the basis function  $f_I(y,b)$  and integrated over the element to obtain

$$\int_0^b E(\bar{y}) f_I(\bar{y},b) d\bar{y} = \left\{ \begin{array}{ll} \int_0^b E(y) f_1(y) dy & (I = 1) \\ \int_0^b E(y) f_2(\bar{y},b) dy & (I = 2) \end{array} \right\} \quad (14)$$

where

$$E(\bar{y}) = \left(1 - \tilde{K}_x M\right) F'' + 2\tilde{K}_x M' F' + K^2 \left\{ \left(1 - \tilde{K}_x M\right)^2 - \left[ \tilde{K}_x^2 + \left(m^2 \pi^2 / K^2 L^2\right) \right] \left(1 - \tilde{K}_x M\right) \right\} F \quad (15)$$

Equation (14) expresses the desired averaging of the residual error, but it does not admit the influence of the boundary conditions. Integration by parts is now incorporated to reduce the order of the second-derivative term in equation (14) and to incorporate the boundary conditions. After integration by parts, equation (14) becomes

$$\begin{aligned} \int_0^b E(\bar{y}) f_I(\bar{y}) d\bar{y} &= \int_0^b \left\{ 3\tilde{K}_x M' F' f_I - \left(1 - \tilde{K}_x M\right) F' f_I + K^2 \left[ \left(1 - \tilde{K}_x M\right)^3 \right. \right. \\ &\quad \left. \left. - \left(1 - \tilde{K}_x M\right) \left[ \tilde{K}_x^2 + \left(m^2 \pi^2 / K^2 L^2\right) \right] \right] F f_I \right\} dy \\ &\quad + F'(b) \left[ 1 - \tilde{K}_x M(b) \right] f_I(b,b) - F'(0) \left[ 1 - \tilde{K}_x M(0) \right] f_I(0,b) \end{aligned} \quad (16)$$

In regard to the first element, equation (8) is used to eliminate  $F'(0)$ ; therefore,

$$F'(0) \left[ 1 - \tilde{K}_x M(0) \right] f_I(0,b) = -iK\beta_{w,\ell} \left[ 1 - \tilde{K}_x M(0) \right]^n \left[ 1 - \tilde{K}_x M(0) \right] F(0) f_I(0,b) \quad (17)$$

Similarly, equation (9) is used to eliminate  $F'(b)$  from the last element so that

$$F'(b) \left[ 1 - \tilde{K}_x M(b) \right] f_I(b, b) = iK\beta_{w,u} \left[ 1 - \tilde{K}_x M(b) \right]^n \left[ 1 - \tilde{K}_x M(b) \right] F(b) f_I(b, b) \quad (18)$$

Substituting equations (10), (11), (17), and (18) into equation (14) leads to the element stiffness matrix. Thus, the contribution to the minimization of the residual error is expressed in matrix form:

$$\int_0^b E(\bar{y}) f_I(\bar{y}, b) d\bar{y} = [A^q] \{\Phi^q\} \quad (19)$$

in which the element stiffness matrix  $[A^q]$  is

$$[A^q] = \begin{bmatrix} a_{11}^q & a_{12}^q \\ a_{21}^q & a_{22}^q \end{bmatrix}$$

the element vector  $\{\Phi^q\}$  contains the values of the function  $F$  at nodes  $q$  and  $q+1$  of the element

$$\{\Phi^q\}^{Tr} = \{F_q, F_{q+1}\}$$

and the superscript  $Tr$  denotes the vector transpose.

Now, equation (19) gives the contribution to the minimization of the residual error due to element  $q$ . The contribution to the residual error due to elements 1 to  $N-1$  is obtained by requiring that at nodes where any two elements are connected, the value of the function  $F(y)$  at that node is the same for both elements. This

requirement is achieved by adding the matrix elements  $a_{11}^{q+1}$  and  $a_{22}^q$  in the global system for any two connected elements. The result is a set of linear matrix equations, which are expressible in the form

$$[A(\beta_{w,u})] \{\Phi\} = \{0\} \quad (20)$$

where

$$[A(\beta_{w,u})] = \begin{bmatrix} a_{11}^1 & a_{12}^1 & & & \\ a_{21}^1 & \bar{a}_{22}^1 & a_{12}^2 & & \\ & a_{21}^2 & \bar{a}_{22}^2 & a_{12}^3 & \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot \\ & & & & a_{21}^{N-1} & a_{22}^{N-1} \end{bmatrix}$$

$$\bar{a}_{22}^q = a_{11}^{q+1} + a_{22}^q$$

$$\{\Phi\} = \{F_1, F_2, F_3, \dots, F_N\}$$

For a nontrivial solution to equation (20), the determinant (det) of the coefficient matrix  $[A(\beta_{w,u})]$  must vanish:

$$\det[A(\beta_{w,u})] = \{0\} \quad (21)$$

Now, the power of the approach used here is that  $\beta_{w,u}$  appears linearly in  $[A(\beta_{w,u})]$ . In fact, the only element in this coefficient matrix which depends on  $\beta_{w,u}$  is the element in the last row and column  $a_{22}^{N-1}$ . The explicit dependence of this element on the unknown admittance can be determined. Thus,  $a_{22}^{N-1}$  is expressible in the form

$$a_{22}^{N-1} = b_{N,N} - \beta_{w,u} d_{N,N}$$

where  $b_{N,N}$  and  $d_{N,N}$  are independent of the unknown admittance  $\beta_{w,u}$ . The coefficient matrix  $[A(\beta_{w,u})]$  is decomposed as follows:

$$[A(\beta_{w,u})] = [B] - \beta_{w,u} [D] \quad (22)$$

Thus,  $[B]$  is obtained from  $[A]$  by replacing  $a_{22}^{N-1}$  by  $b_{N,N}$ , whereas  $[D]$  has a single nonzero coefficient in its last row and column.

Substituting equation (22) into (20) leads to the linear eigenvalue problem

$$[B]\{\Phi\} = \beta_{w,u} [D]\{\Phi\} \quad (23)$$

Because  $[B]$  is tridiagonal and  $[D]$  has a nonzero element in its last row and column only, equation (23) is fairly easy to solve. In this analysis, equation (23) is solved by using a special adaptation of the Gaussian elimination procedure that employs row pivoting and row scaling to maintain stability. This procedure transforms equation (23) into a new system of equations which are expressible in the form

$$[\bar{B}]\{\Phi\} = \beta_{w,u} [\bar{D}]\{\Phi\} \quad (24)$$

$$[\bar{B}] = \begin{bmatrix} \bar{b}_{1,1} & \bar{b}_{1,2} & \bar{b}_{1,3} & & & \\ & \bar{b}_{2,2} & \bar{b}_{2,3} & \bar{b}_{2,4} & & \\ & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & \cdot \\ & & & & \bar{b}_{N-1,N-1} & \bar{b}_{N-1,N} \\ & & & & & \bar{b}_{N,N} \end{bmatrix}$$

$$[\bar{D}] = \begin{bmatrix} 0 & 0 & & & & \\ & 0 & 0 & & & \\ & & \cdot & \cdot & & \\ & & & \cdot & \cdot & \\ & & & & \cdot & \bar{d}_{N-1,N} \\ & & & & & \bar{d}_{N,N} \end{bmatrix}$$

Since both  $[\bar{B}]$  and  $[\bar{D}]$  are upper triangular matrices, equation (23) can be solved directly for the admittance  $\beta_{w,u}$  and eigenvector  $\{\Phi\}$ . The last equation in this system reads

$$\left( \bar{b}_{N,N} - \beta_{w,u} \bar{d}_{N,N} \right) F_N = 0 \quad (25)$$

The solution to this equation is

$$\beta_{w,u} = \bar{b}_{N,N} / \bar{d}_{N,N} \quad (26)$$

It is instructive to note that the value of  $\beta_{w,u}$  given by equation (26) is unique and is the only value which makes the determinant of the coefficient matrix  $[A(\beta_{w,u})]$  vanish. (See eq. (21).) Further, the value of  $F_N$  in equation (25) is set to unity and the remaining nodal values of the function  $F(y)$  are obtained from equation (24) by back substitution.

## RESULTS AND DISCUSSION

In order to verify the numerical technique, results from the method are compared both with exact solutions that can be obtained for a constant mean-flow profile and with results of a Runge-Kutta integration technique for cases involving shear. To this end, only the lowest order cross mode is considered ( $m = 0$ ), and continuity of particle velocity ( $n = 1$ ) is employed as the boundary condition along the duct walls. Further, only 40 locations are employed in the finite-element discretization across the duct. The choice of this particular combination of parameters was felt to be sufficient to obtain confidence in the numerical technique.



## Uniform Flow

In the case of a constant mean-flow profile, the admittance  $\beta_{w,u}$  can be computed from the equation

$$\beta_{w,u} = \left[ \lambda^2 \tan \lambda h + iK\lambda\beta_{w,\ell} \left( 1 - \tilde{K}_x M \right)^n \right] / \left[ -iK\lambda \left( 1 - \tilde{K}_x M \right)^n - K^2 \beta_{w,\ell} \left( 1 - \tilde{K}_x M \right)^{2n} \left( 1 - \tilde{K}_x M \right)^{2n} \tan \lambda h \right] \quad (27)$$

where the eigenvalue  $\lambda$  is related to the decay rate by the equation

$$\lambda^2 = \left\{ \left( 1 - M\tilde{K}_x \right)^2 - \left[ \tilde{K}_x^2 + \left( m^2 \pi^2 / K^2 L^2 \right) \right] \right\} K^2$$

Equation (27) is easily derived by finding the general solution to equation (7) for a constant mean-flow profile and then applying the boundary conditions (eqs. (8) and (9)). Thus, for specified values of  $K$ ,  $h$ ,  $\tilde{K}_x$ ,  $L$ ,  $m$ , and  $\beta_{w,\ell}$ , equation (27) can be solved directly for the grazing-flow admittance.

The  $\tilde{K}_x$  values corresponding to a hard-wall duct are determined by setting both  $\beta_{w,u}$  and  $\beta_{w,\ell}$  to 0 in equation (27) and then solving this equation for  $\tilde{K}_x$ . The  $\tilde{K}_x$  values corresponding to the first- and third-order mode in a hard-wall duct have been computed from equation (27) with  $K$  and  $h$  both set to unity. These values are presented to two decimal places in table I for a range of constant mean-flow profiles. The values of  $\beta_{w,\ell}$ ,  $K$ ,  $h$ ,  $M$ , and  $\tilde{K}_x$  given in table I were used as input into equation (20) so that the admittance  $\beta_{w,u}$  could be determined. In all cases, the admittance value determined from equation (20) agreed with the exact value of 0 to within four decimal places.

The present method is also applicable to ducts for which the bottom wall is not hard (i.e.,  $\beta_{w,\ell} \neq 0$ ) but has a finite value. Values of  $\beta_{w,u}$  computed from equation (27) for  $\beta_{w,\ell} = 0 + 1.0i$ , with  $K$  and  $h$  both set to unity, and for a range of  $M$  and  $\tilde{K}_x$  values are presented in table II. The values of  $\beta_{w,\ell}$ ,  $M$ ,  $\tilde{K}_x$ ,  $K$ , and  $h$  given in table II were used as input into equation (20) and thus the admittance could be determined. In all cases, the admittance  $\beta_{w,u}$  determined from equation (20) agreed with those in table II to the number of decimal places shown in the table.

## Shear Flow

When there is shear so that  $M'(y)$  is not 0, an exact expression for the admittance is apparently impossible. Therefore, the effort was made to evaluate the validity of the current numerical method in the presence of shear by comparing results to that of a Runge-Kutta integration technique (ref. 1).

TABLE I.- AXIAL PROPAGATION CONSTANT FOR FIRST- AND THIRD-ORDER  
MODE IN HARD-WALL DUCT

$$[K = 1; \beta_{w,u} = \beta_{w,l} = 0 + 0i; h = 1]$$

M	$\tilde{K}_x$ for first-order mode	$\tilde{K}_x$ for third-order mode
-0.5	2.00 + 0.00i	0.67 + 7.13i
-.4	1.67 + .00i	.48 + 6.75i
-.3	1.43 + .00i	.33 + 6.49i
-.2	1.25 + .00i	.21 + 6.33i
-.1	1.11 + .00i	.10 + 6.23i
.0	1.60 + .00i	.00 + 6.20i
.1	.91 + .00i	-.10 + 6.23i
.2	.83 + .00i	-.21 + 6.33i
.3	.77 + .00i	-.33 + 6.49i
.4	.71 + .00i	-.48 + 6.75i
.5	.67 + .00i	-.67 + 7.13i

TABLE II.- EXACT VALUE OF  $\beta_{w,u}$  FOR SOFT-WALL DUCT

$$[K = 1; \beta_{w,l} = 0 + 1.0i; h = 1]$$

M	$\tilde{K}_x$	Exact admittance, $\beta_{w,u}$
-0.5	2.29 + 1.83i	0.519 - 0.967i
-.4	1.99 + 1.76i	.662 - 1.151i
-.3	1.767 + 1.709i	.860 - 1.334i
-.2	1.598 + 1.675i	1.135 - 1.500i
-.1	1.465 + 1.655i	1.512 - 1.614i
.0	1.356 + 1.650i	2.000 - 1.600i
.1	1.263 + 1.655i	2.545 - 1.349i
.2	1.182 + 1.675i	2.967 - .734i
.3	1.107 + 1.709i	3.041 - .061i
.4	1.035 + 1.761i	2.738 + .561i
.5	.958 + 1.835i	2.255 + 1.899i

The  $\tilde{K}_x$  values presented in table III were computed from a Runge-Kutta integration technique identical to that used in reference 1. Values of  $\tilde{K}_x$  shown in the

TABLE III.- AXIAL PROPAGATION CONSTANT FOR FIRST-ORDER MODE  
IN HARD-WALL DUCT FOR CONSTANT-GRADIENT SHEAR FLOW

$$[K = 2\pi; \beta_{w,l} = \beta_{w,u} = 0 + 0i; h = 2]$$

$M_o$	$\tilde{K}_x$ for first-order mode
-0.3	7.880 + 0.000i
-.1	6.655 + .000i
.1	6.010 + .000i
.3	5.612 + .000i

table correspond to the first-order mode in a hard-wall duct ( $\beta_{w,l} = \beta_{w,u} = 0 + 0.00i$ ) with  $h = 2$  and  $K = 2\pi$ . A constant-gradient laminar shear flow was employed for which

$$M(y) = \begin{cases} 2M_o y/h & (0 < y < h/2) \\ 2M_o [1 - (y/h)] & (h/2 < y < h) \end{cases} \quad (28)$$

where  $M_o$  is the centerline Mach number. The values of  $h$ ,  $\tilde{K}_x$ ,  $M_o$ ,  $\beta_{w,l}$ , and  $K$  shown in table III were input into equation (20) and thus the admittance  $\beta_{w,u}$  could be determined. In all cases, values of  $\beta_{w,u}$  determined from equation (20) were 0 up to four decimal places.

#### CONCLUDING REMARKS

A new method has been developed for calculating acoustic-liner admittance in a rectangular duct with grazing flow. The approach is based upon a finite-element discretization of the acoustic field and a reposing of the unknown admittance value as a linear eigenvalue problem on the admittance value. Gaussian elimination is employed to solve this eigenvalue problem. Unlike existing methods that are based upon the integration of an ordinary differential equation and are thus limited to mean flows with one-dimensional boundary layers, the present method is extendable to mean flows with two-dimensional boundary layers as well.

Admittance values determined from the method were compared with exact solutions obtained for a constant mean-flow profile, with excellent comparisons being obtained.

In the presence of shear, the results of the method were compared with results of a Runge-Kutta integration technique, with good comparisons again being obtained. The excellent results obtained from the method for the various flow profiles considered here motivate its extension to include grazing flows with two-dimensional boundary layers. Such an extension should now be formulated.

Langley Research Center  
National Aeronautics and Space Administration  
Hampton, VA 23665  
May 22, 1984

#### REFERENCES

1. Mungur, P.; and Gladwell, G. M. L.: Acoustic Wave Propagation in a Sheared Fluid Contained in a Duct. *J. Sound & Vib.*, vol. 9, no. 1, Jan. 1969, pp. 28-48.
2. Armstrong, D. L.; Beckemeyer, R. J.; and Olsen, R. F.: Impedance Measurements of Acoustic Duct Liners With Grazing Flow. 87th Meeting of the Acoustical Society of America (New York), Apr. 1974.
3. Unruh, J. F.; and Eversman, W.: The Transmission of Sound in an Acoustically Treated Rectangular Duct With Boundary Layer. *J. Sound & Vib.*, vol. 25, no. 3, Dec. 1972, pp. 371-382.



1. Report No. NASA TP-2310		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle A NEW METHOD FOR DETERMINING ACOUSTIC-LINER ADMITTANCE IN A RECTANGULAR DUCT WITH GRAZING FLOW FROM EXPERIMENTAL DATA				5. Report Date July 1984	
				6. Performing Organization Code 505-31-33-13	
7. Author(s) Willie R. Watson				8. Performing Organization Report No. L-15779	
9. Performing Organization Name and Address  NASA Langley Research Center Hampton, VA 23665				10. Work Unit No.	
				11. Contract or Grant No.	
				13. Type of Report and Period Covered Technical Paper	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546				14. Sponsoring Agency Code	
15. Supplementary Notes					
16. Abstract  A new method is developed for determining acoustic-liner admittance in a rectangular duct with grazing flow. The approach is to measure the axial propagation constant, cross-mode order, and mean-flow profile. These measured data are then input into an analytical program which determines the unknown admittance value. The analytical program is based upon a finite-element discretization of the acoustic field and a reposing of the unknown admittance value as a linear eigenvalue problem on the admittance value. Gaussian elimination is employed to solve this eigenvalue problem. Unlike existing methods, the method used here is extendable to grazing flows with boundary layers in both transverse directions of an impedance tube (or duct). Predicted admittance values computed by this method are compared both with exact values that can be obtained for uniform mean-flow profiles and with those from a Runge-Kutta integration technique for cases involving a one-dimensional boundary layer.					
17. Key Words (Suggested by Author(s)) Admittance prediction Eigenvalue problems Numerical technique Grazing-flow admittance Finite-element approach			18. Distribution Statement Unclassified - Unlimited  Subject Category 71		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 17	22. Price A02		



National Aeronautics and  
Space Administration

Washington, D.C.  
20546

Official Business  
Penalty for Private Use, \$300

THIRD-CLASS BULK RATE

Postage and Fees Paid  
National Aeronautics and  
Space Administration  
NASA-451



**NASA**

POSTMASTER: If Undeliverable (Section 158  
Postal Manual) Do Not Return

---